Math 115

Spring 2018

Lecture 16

$? \; a^2 + b^2 = c^2 \; ?$
$?$
$y = mx + b \quad d = rt$

Class Quiz

① find slope of a line that contains
(5, -3) and (-2, 3) $m = \dfrac{-3-3}{5-(-2)} = \boxed{\dfrac{-6}{7}}$

② find eqn of a line with slope $\frac{2}{3}$ that
contains the origin. $y - y_1 = m(x - x_1)$
$y - 0 = \frac{2}{3}(x - 0) \quad \boxed{y = \frac{2}{3}x}$

③ find <u>Slope</u> of a line that is <u>perpendicular</u> to
the line $4x + 3y = 6$. $\boxed{y = \frac{-4}{3}x + 2}$
$3y = -4x + 6$ opposite Reciprocal
$\boxed{m = \frac{3}{4}}$

Ch.4 : Working with Polynomial
Exponential Rules

① x^n $x \rightarrow$ base , $n \rightarrow$ Exponent (Integer exponent)

$$x^n = \underbrace{x \cdot x \cdot x \cdots x}_{n \text{ times}}$$

$$x^3 = x \cdot x \cdot x \quad , \quad (2x)^4 = (2x) \cdot (2x) \cdot (2x) \cdot (2x)$$

$$(-3x^2y)^5 = \underbrace{(-3x^2y) \cdot (-3x^2y) \cdots (-3x^2y)}_{5 \text{ times}}$$

② $x^1 = x$

$$5^1 = 5 \quad , \quad (-8)^1 = -8 , \quad (4x^3)^1 = 4x^3$$

$$\left(\frac{-3}{5}\right)^1 = \frac{-3}{5} \cdot \left(-2x^4y^5\right)^1 = -2x^4y^5$$

③ $x^0 = 1$, $x \neq 0$

$$5^0 = 1 \quad , \quad (-24)^0 = 1 , \quad (4x^2)^0 = 1 \quad x \neq 0$$

$$(7x^3y^2)^0 = 1 \quad , \quad \left(\frac{2x}{5y^3}\right)^0 = 1 \quad , \quad (2x^2+10)^0 = 1$$

④ $x^m \cdot x^n = x^{m+n}$

$x^3 \cdot x^5 = x^{3+5} = x^8$

$x^6 \cdot x^9 \cdot x^{\boxed{1}} = x^{6+9+1} = x^{16}$

$(-4x^3)^{12} \cdot (-4x^3)^{18} = (-4x^3)^{12+18} = (-4x^3)^{30}$

$\left(\dfrac{2x}{5y^3}\right)^{7} \cdot \left(\dfrac{2x}{5y^3}\right)^{10} = \left(\dfrac{2x}{5y^3}\right)^{17}$

⑤ $(x^m)^n = x^{m \cdot n}$

$(x^3)^5 = x^{3 \cdot 5} = x^{15}$

$(x^4)^2 = x^{4 \cdot 2} = x^8$

$(x^{\boxed{6}})^{10} \cdot x^5 = x^{\boxed{60}} \cdot x^{\boxed{5}} = \boxed{x^{65}}$

$(x^8)^3 \cdot (x^3)^7 = x^{24} \cdot x^{21}$

$= x^{24+21} = \boxed{x^{45}}$

⑥ $(xy)^n = x^n y^n$

$(2x)^3 = 2^3 x^3 = \boxed{8x^3}$

$(-3x^2)^4 = (-3)^4 (x^2)^4 = \boxed{81x^8}$

$(-4x^6 y^7)^3 = (-4)^3 (x^6)^3 (y^7)^3$

$= \boxed{-64x^{18} y^{21}}$

Simplify

① $10^1 - (-10)^0 = 10 - 1 = \boxed{9}$

② $\left(\frac{3}{2}\right)^3 - \left(\frac{-1}{4}\right)^2 = \frac{27 \cdot 2}{8 \cdot 2} - \frac{1}{16} = \frac{54}{16} - \frac{1}{16} = \frac{54-1}{16}$

$= \boxed{\frac{53}{16}}$

③ $x^{12} \cdot (x^8)^5 = x^{12} \cdot x^{40} = \boxed{x^{52}}$

④ $(-2x^6 y^4)^5$

$= (-2)^5 (x^6)^5 (y^4)^5 = \boxed{-32x^{30} y^{20}}$

⑦ $\dfrac{x^m}{x^n} = x^{m-n}$

$\dfrac{x^7}{x^3} = x^{7-3} = \boxed{x^4}$

$\dfrac{x^9}{x^8} = x^{9-8} = x^1 = \boxed{x}$

$\dfrac{(x^6)^5}{(x^{10})^3} = \dfrac{x^{6\cdot5}}{x^{10\cdot3}} = \boxed{\dfrac{x^{30}}{x^{30}}} = x^{30-30} = x^0$

$\rightarrow = \boxed{1}$

Your turn:

Simplify:

① $\dfrac{(-3x^6)^3 \cdot x^2}{(x^4)^5 \cdot x^6}$ ... $(-3)^3\, x^{6\cdot3} \cdot x^2 = -27\, x^{18} x^2$

$= \boxed{-27\, x^{20}}$

② $\dfrac{(x^4)^5 \cdot x^6}{(x^5)^2 \cdot (x^2)^5 (x^3)^2} = \dfrac{x^{20} \cdot x^6}{x^{10} \cdot x^{10} \cdot x^6}$

$= \dfrac{x^{26}}{x^{26}} = \boxed{1}$

⑧ $\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$\left(\dfrac{x}{5}\right)^2 = \dfrac{x^2}{5^2} = \dfrac{x^2}{25}$

$\left(\dfrac{3x}{y^2}\right)^4 = \dfrac{(3x)^4}{(y^2)^4} = \dfrac{3^4 x^4}{y^{2\cdot4}} = \boxed{\dfrac{81x^4}{y^8}}$

$\left(\dfrac{2x^3}{5y^6}\right)^3 = \dfrac{2^3(x^3)^3}{5^3(y^6)^3} = \dfrac{8x^9}{125y^{18}}$

⑨ $x^{-n} = \dfrac{1}{x^n}$

$x^{-3} = \boxed{\dfrac{1}{x^3}}$, $10^{-2} = \dfrac{1}{10^2} = \boxed{\dfrac{1}{100}}$, $2^{-1} = \dfrac{1}{2^1} = \dfrac{1}{2}$

$\left(x^5\right)^{-4} = x^{5\cdot(-4)} = x^{-20} = \boxed{\dfrac{1}{x^{20}}}$

Simplify

$\left(x^{-6}\right)^5 \cdot \left(x^{-4}\right)^{-3} = x^{-30} \cdot x^{12} = x^{-30+12}$

$= x^{-18} = \boxed{\dfrac{1}{x^{18}}}$

⑩ $\dfrac{x^{-n}}{y^{-m}} = \dfrac{y^m}{x^n}$

$\dfrac{x^{-3}}{y^{-5}} = \dfrac{y^5}{x^3}$, $\dfrac{\boxed{x^{-4}}y^6}{x^3\boxed{y^{-6}}} = \dfrac{y^6}{x^3}\dfrac{y^6}{x^4}$

$\dfrac{(x^5)^{-2}\cdot(y^4)^6}{(x^{-3})^{-1}\cdot(y^{-2})^3} = \dfrac{x^{-10}\cdot y^{24}}{x^3\cdot y^{-6}}$ $= \boxed{\dfrac{y^{12}}{x^7}}$

$24-(-6)=$
$24+6=30$
$= \dfrac{y^6\cdot y^{24}}{x^3\cdot x^{10}} = \boxed{\dfrac{y^{30}}{x^{13}}}$

⑪ $\left(\dfrac{x}{y}\right)^{-n} = \left(\dfrac{y}{x}\right)^n$

$\left(\dfrac{2}{x}\right)^{-3} = \left(\dfrac{x}{2}\right)^3 = \dfrac{x^3}{2^3} = \boxed{\dfrac{x^3}{8}}$

$\left(\dfrac{x^4}{y^7}\right)^{-5} = \left(\dfrac{y^7}{x^4}\right)^5 = \dfrac{(y^7)^5}{(x^4)^5} = \boxed{\dfrac{y^{35}}{x^{20}}}$

$\left(\dfrac{2x^3}{3y^5}\right)^{-2} = \left(\dfrac{3y^5}{2x^3}\right)^2 = \dfrac{3^2(y^5)^2}{2^2(x^3)^2} = \boxed{\dfrac{9\,y^{10}}{4\,x^6}}$

Simplify

Hint: Always do inside of groups.

$$\left(\frac{-2x^{-4}}{3y^{-7}}\right)^{-3} = \left(\frac{-2\ y^7}{3\ x^4}\right)^{-3} = \left(\frac{3x^4}{-2y^7}\right)^3$$

$$= \frac{3^3\left(x^4\right)^3}{(-2)^3\left(y^7\right)^3} = \frac{27x^{12}}{-8\ y^{21}} = \boxed{\frac{-27x^{12}}{8\ y^{21}}}$$

---

find two complementary angles such that the difference of 3 times one of them and 4 times the other one is −220°.

$$\begin{array}{l} 4\begin{cases} x + y = 90 \\ 3x - 4y = -220 \end{cases} \Rightarrow \begin{cases} 4x + 4y = 360 \\ 3x - 4y = -220 \end{cases} \\ \hline \qquad\qquad 7x \qquad = 140 \\ \qquad\qquad x = 20 \end{array}$$

$$20° \ \& \ 70°$$

find two supplementary angles such that one of them is 30° more than twice the other one.

$$\begin{cases} x + y = 180 \\ x = 2y + 30 \end{cases}$$

$$2y + 30 + y = 180$$
$$3y = 150$$
$$y = 50$$

50° & 130°

---

30 TKTS was purchased.
Total cost → $199
Students Pay → $5
Parents Pay → $12
How many of each?

S → Students
P → Parents

$$-5 \begin{cases} S + P = 30 \\ 5S + 12P = 199 \end{cases}$$

$$\begin{cases} -5S - 5P = -150 \\ 5S + 12P = 199 \end{cases}$$
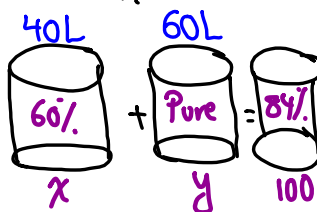$$\overline{}$$
$$7P = 49$$
$$\boxed{P = 7}$$

7 parents
&
23 Students

Lisa needs 100 liters of 84% acid Solution. She has unlimited supply of 60% acid Solution and pure acid Solution. How many liters of each should she mix to obtain what she needs?

40L   60L

60% + Pure = 84%

x y 100

$$\begin{cases} x + y = 100 \\ 100(60\% \cdot x + 100\% \cdot y = 84\% \cdot 100) \end{cases}$$

$$\div 10 \begin{cases} x + y = 100 \\ 60x + 100y = 84 \cdot 100 \end{cases} \quad \div 2 \begin{cases} x + y = 100 \\ 6x + 10y = 840 \end{cases}$$

$$-3 \begin{cases} x + y = 100 \\ 3x + 5y = 420 \end{cases} \rightarrow \begin{cases} -3x - 3y = -300 \\ 3x + 5y = 420 \end{cases}$$

$$2y = 120$$
$$\boxed{y = 60}$$

$\boxed{\text{60 L of Pure & 40 L of 60% Acid}}$

Leo made $2000 investment in 2 accounts. One paid 3% Simple interest, the other one paid 5% Simple interest. After 1 Year, he earned $84 in total interest. How much per account?

$$\begin{cases} x + y = 2000 \\ 3\% x \cdot 1 + 5\% y \cdot 1 = 84 \end{cases} \quad 100 \begin{cases} x + y = 2000 \\ .03x + .05y = 84 \end{cases}$$

$$-3 \begin{cases} x + y = 2000 \\ 3x + 5y = 8400 \end{cases} \rightarrow \begin{cases} -3x - 3y = -6000 \\ 3x + 5y = 8400 \end{cases}$$

$$2y = 2400$$
$$y = 1200$$

$\boxed{\$1200 @ 5\% \text{ rate} \\ \& \\ 800 @ 3\% \text{ rate}}$

Jake deposited $5000 in two accounts. One paid 8% Simple interest, and the other one paid 5% Simple interest. After one Year, interest earned from 8% account was $270 more than interest earned from 5% account. Find how much per account?

$$\begin{cases} x + y = 5000 \\ 8\% \cdot x \cdot 1 = 5\% \cdot y \cdot 1 + 270 \end{cases}$$

$\underbrace{8\% \cdot x \cdot 1}_{\text{Interest}} \quad \underbrace{5\% \cdot y \cdot 1}_{\text{Interest}}$

$$\begin{cases} x + y = 5000 \\ 100 \Big[.08x - .05y = 270 \end{cases}$$

$$5\begin{cases} x + y = 5000 \\ 8x - 5y = 27000 \end{cases} \Rightarrow \begin{cases} 5x + 5y = 25000 \\ 8x - 5y = 27000 \end{cases}$$

$$13x = 52000$$

$$\boxed{x = 4000}$$

$\boxed{\$4000 @ 8\%, \$1000 @ 5\%}$